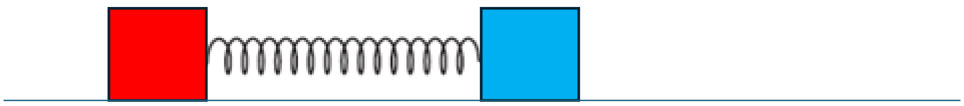
Teacher notes
Topic A

An unexpected motion.

Two blocks, each of mass 2.0 kg, are joined by a massless spring of natural length 0.40 m. The system is on a frictionless horizontal table and is initially at rest. The spring has its natural length.



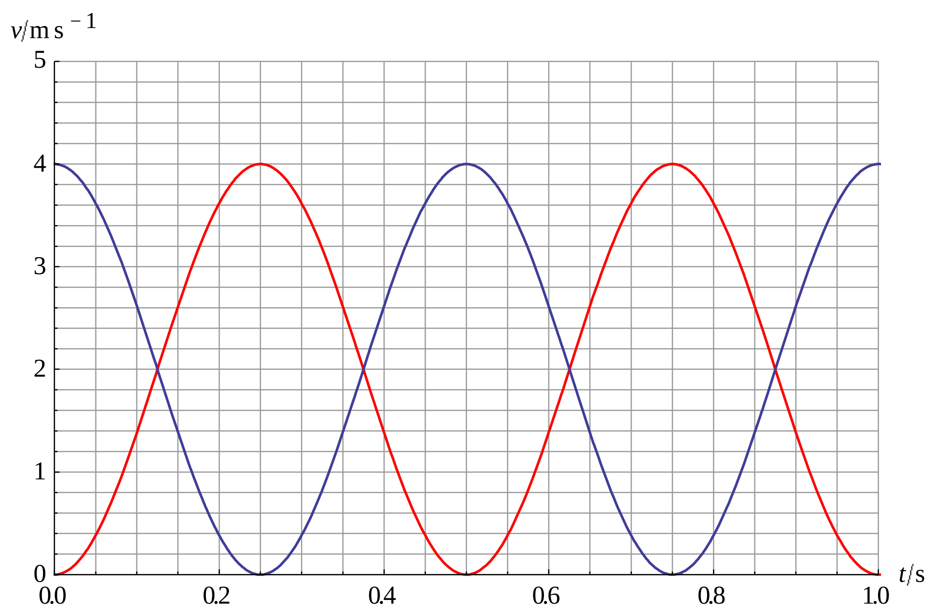
The blue block is given a very sharp push and moves with speed 4.0 m s^{-1} to the right.

(This can be investigated with an air track and appropriate sensors. The motion is very unexpected.)

(a) Determine

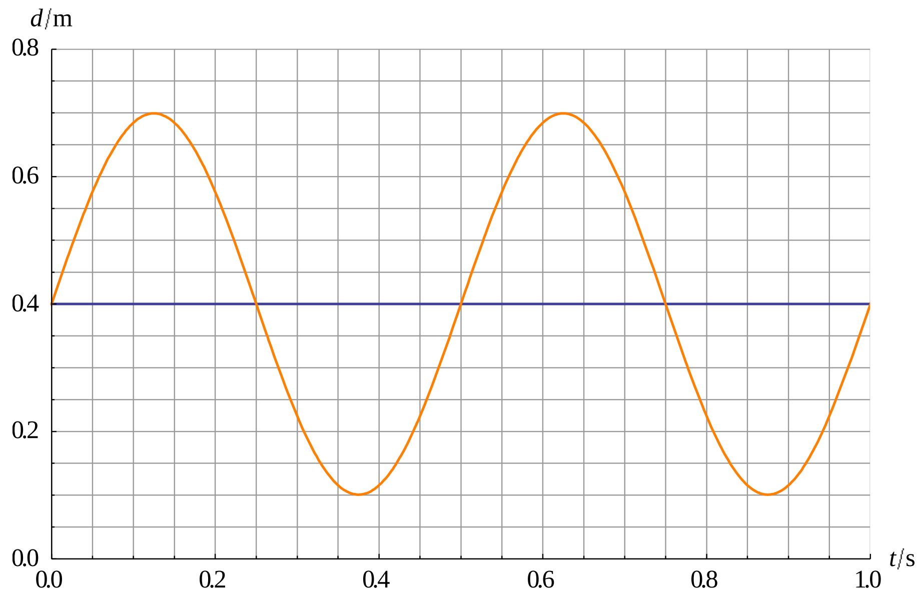
(i) the impulse delivered to the mass-spring system,
(ii) the total energy of the mass-spring system.

(b) The speeds of the two blocks (red and blue) are given by graph 1.



Graph 1

The distance *d* between the two blocks is given by graph 2.



Graph 2

(i) Describe the motion of the blocks.

(ii) Explain why at the times when the two blocks have a common speed that common speed is 2.0 m s^{-1}.

(iii) Estimate the spring constant, explaining your reasoning.

Answers

(a)

(i) The blue block receives the impulse very quickly so there is no time for the spring to extend or the red block to move. Hence $J = mv = 2.0 \times 4.0 = 8.0\ \text{N s}$.

(ii) For the same reason, the total energy after the push is $E = \frac{1}{2}mv^2 = \frac{1}{2} \times 2.0 \times 4.0^2 = 16\ \text{J}$. There are no dissipative forces acting past that point and so the total energy remains at 16 J.

(b)

(i) The blue block moves off extending the spring and so suffers a decelerating force (the spring tension) that brings the speed to zero instantaneously. The tension accelerates the red block to the same maximum speed of 4.0 m s^{-1} in the same time that the blue block comes to rest. The distance between the blocks is oscillating in SHM with amplitude 0.30 m and period 0.50 s; therefore, the maximum compression and maximum extension of the spring are 0.30 m. The distance is a maximum and minimum at those times when the speeds of the blocks are equal. (See also the Teacher Note "An energy and momentum problem".)

(ii) The total momentum after the push is 8.0 N s. There are no external forces after this point and so momentum is conserved. Hence, when the speeds are equal, $mv = 2mu \Rightarrow u = \frac{v}{2} = 2.0\ \text{m s}^{-1}$.

(iii) From the graph, the maximum compression and maximum extension of the spring is 0.30 m and this happens at times when the speeds of the blocks are the same. The total energy at these times is then $E = \frac{1}{2} \times 2.0 \times 2.0^2 + \frac{1}{2} \times 2.0 \times 2.0^2 + E_e = 16\ \text{J}$. Hence $E_e = 8.0\ \text{J}$. Finally, $E_e = \frac{1}{2}kx^2 = \frac{1}{2}k \times 0.30^2 = 8.0\ \text{J}$. Hence, $k = \frac{16}{0.30^2} \approx 180\ \ \text{N m}^{-1}$.

PS The distance travelled by each block separately is shown in the graph.